

The ecology of gaze shifts

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Abstract

We present a phenomenological model for the generation of human visual scanpaths. Successions of saccadic eye movements are treated as realizations of a stochastic jump process in a random quenched salience field. Efficiency of the process is defined in terms of convergence properties of the time-dependent probability of fixating a region in the visual environment. Based on the assumption that the visual system minimizes the typical time needed to process a visual scene, our theory predicts that scanpaths are geometrically similar to a prominent class of random walks known as Lévy flights. The theory is well confirmed by psychophysical experiments. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The visual system of primates achieves highest resolution in the fovea. In the periphery, visual resolution rapidly decreases. The loss of visual acuity is compensated by a succession of rapid eye movements known as saccades. Saccades actively reposition the center of gaze in order to foveate regions of interest in the visual environment. The succession of gaze shifts is referred to as a scanpath. Two typical scanpaths of a subject scanning a party scene are depicted in Fig. 1. A histogram of saccadic magnitudes computed from the paths is shown as well. The main features of

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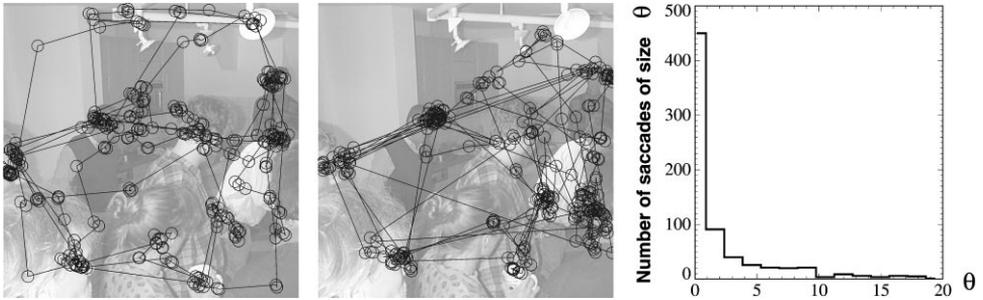


Fig. 1. Left, Center: Two typical scanpaths on different trials by the same subject. Each scanpath consists of approximately 350 saccades. Right: Saccadic magnitude histogram calculated from the scanpaths depicted. θ denotes saccadic magnitude in degrees of visual angle.

human scanpaths can be extracted from this figure:

- Fixations accumulate in visually salient regions.
- Scanpaths connecting salient locations differ between trials.
- Short saccades are more frequent than long ones.

Statistically, a distance-dependent saccadic magnitude distribution implies that the visual system requires a certain amount of time to scan the entire visual field. That is, if an ensemble of scanpaths started at a certain location in the visual field, it requires a certain amount of time T to distribute the current positions of gaze evenly in visual space. This lead us to investigate whether human saccadic magnitude distributions and the scanpath geometry they entail can be understood in terms of a temporal optimization principle, especially when operating in a natural environment. To this end we introduce a simple model in which scanpaths are interpreted as realizations of a stochastic process with non-local transition probabilities in a random salience environment. The model is related to random walks in quenched disordered potentials (for a review see [1]), frequently encountered in various fields of physics such as statistical mechanics [4] and chaotic systems [3].

Although probabilistic approaches have been applied to various problems in eye movement research, such as the perception of ambiguous figures [8], the specific functional form of saccadic magnitude distributions has attracted surprisingly little attention. We show that this form has a profound impact on the geometry of scanpaths. Furthermore, our analysis suggests that visual performance under natural conditions is primarily determined by the long-tail behavior of saccadic magnitude probability. A most efficient way of generating scanpaths follows a power law in the frequency of occurrence of shift magnitudes. Power laws in saccadic magnitude distributions relate eye movements to a prominent class of random walks known as Lévy flights. Lévy flights play a role in systems ranging from global climate fluctuations [2] to animal foraging behavior [9]. The Lévy flight nature of human scanpaths predicted by our theory is well confirmed by our psychophysical experiments.

2. Concepts

Consider a random walk defined by a symmetric single-step probability density $p(x)$. At discrete time step t the walker is located at $X = \sum_i^t x_i$ with probability density $P(X, t) = \langle \delta(X - \sum_i^t x_i) \rangle$. Initially, the walker is located at the origin, $P(X, 0) = \delta(X)$. The expression for $P(X, t)$ becomes particularly handsome in Fourier-space

$$\tilde{P}(k, t) = \tilde{p}(k)^t. \tag{1}$$

The quantities $\tilde{P}(k, t)$, $\tilde{p}(k)$ denote the Fourier transforms of $P(X, t)$ and $p(x)$, respectively. Since $|\tilde{p}(k)| \leq 1$, the long-time behavior of $\tilde{P}(k, t)$ is determined by $\tilde{p}(k)$ at low k , corresponding to the behavior of $p(x)$ as $x \rightarrow \infty$. If $p(x) \propto x^{-(1+f)}$, with $0 < f < 2$ then $\tilde{p}(k)$ is non-analytic at $k = 0$ and $\tilde{P}(k, n) \approx (1 - D|k|^\mu)^t$ with $\mu = f$, D being a constant. Thus,

$$\tilde{P}(k, t) \approx e^{-D|k|^\mu t} \quad \text{and} \quad P(x, t) \propto \int dk e^{ikx - D|k|^\mu t}. \tag{2}$$

This well-known result was first derived by Lévy in the 1920s, a more rigorous proof can be found in [1]. Eq. (2) defines the Lévy stable laws of index μ . If $p(x)$ decays faster than x^{-3} ($f > 2$), $\tilde{p}(k)$ is analytic at $k = 0$, the exponent μ locks onto the value of 2 and Eq. (2) reflects the central limit theorem, $P(X, t)$ is Gaussian, and the walk is diffusive. On the other hand, if $0 < \mu < 2$ the second moment of $p(x)$ is infinite and the process is super-diffusive. If for example $p(x) \propto x^{-2}$, then $P(X, t) \propto Dt/(Dt^2 + X^2)$, known as the Cauchy distribution. Geometrically, diffusive walks possess a characteristic length scale (defined by the variance of the single-step distribution), whereas Lévy flights are scale free due to their lack of a finite variance. The impact of the long-tail behavior of the single-step distribution $p(x)$ is depicted in Fig. 2. Although, in either case the single-step distributions look similar at first glance, the path geometries differ considerably.

If random walks occur on a large but finite support of size L the convergence time of $P(X, t)$ to the stationary state is related to the relaxation properties of the lowest mode $k_0 = 2\pi/L$ (provided that $L \gg D^{1/\mu}$):

$$T = \frac{1}{D|k_0|^\mu} = \frac{1}{D} \left(\frac{L}{2\pi} \right)^\mu. \tag{3}$$

This quantity will be of interest in the following. Note that if $\mu = 2$ then $T \propto L^2$ as expected for regular diffusion processes.

2.1. The model

The dynamical quantity of interest in our model is the probability density $p(x, t)$ of fixating a location x at discrete times $t = 1, 2, 3 \dots$ in a visual field of size L . The transition probability density of shifting the gaze from a point y to x is defined as the product of a random quenched salience field $s(x) > 0$ and a term $f(|x - y|)$ the

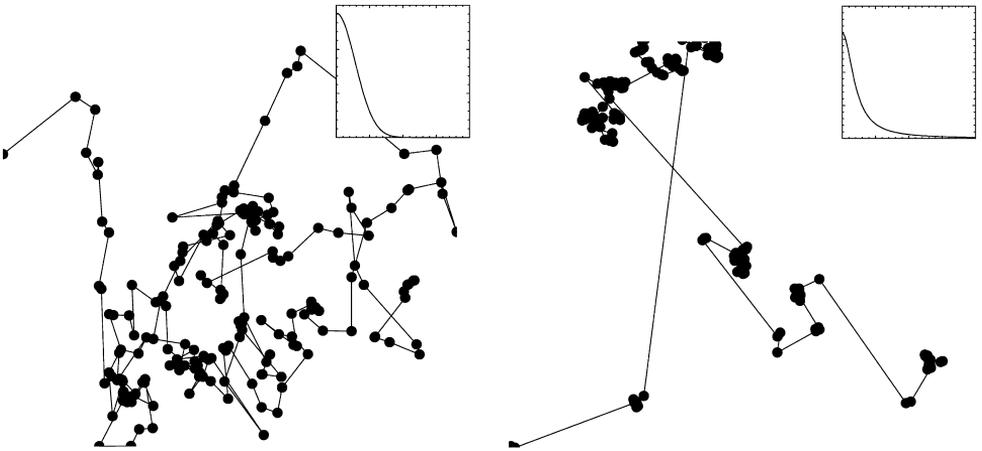


Fig. 2. Left: Regular diffusion process generated from a single-step distribution with finite second moment. Right: A scale free Lévy flight generated from a single-step distribution with an algebraic tail $p(x) \propto x^{-2}$. Insets depict the corresponding single-step distributions. Note that in both cases these functions are monotonically decreasing and similar in shape, yet the corresponding walks differ considerably in their geometrical structure.

interpretation of which will be given below:

$$\rho(x|y) \propto s(x)f(|x-y|). \quad (4)$$

The factor $s(x)$ in Eq. (4) quantifies the salience at the target location x . Here, $s(x)$ is interpreted as the probability of attracting the gaze. The salience field is defined as

$$s(x) = \exp(-\beta V(x)). \quad (5)$$

An ensemble of potentials $V(x)$ is drawn from a Gaussian distribution

$$p[V] \propto e^{-\int dx dy V(x) C^{-1}(|x-y|) V(y)} \quad (6)$$

with covariance C . We considered potentials with different types of covariance. In particular, we chose covariances with algebraic tails in the large $|x-y|$ regime. Second-order statistics of this kind were discovered in ensembles of natural images [7]. The potential is normalized to yield an average salience of $\langle s \rangle = 1/L$. The parameter β controls the magnitude variation within the salience field.

The function $f(|x-y|)$ in Eq. (4) is interpreted as the probability of generating a saccade of magnitude $|x-y|$, averaged over the salience field ensemble. We chose $f(|x-y|)$ to be monotonically decreasing, reflecting the structure of experimentally determined saccadic magnitude histograms, see Fig. 1. Two classes of systems are compared. On one hand $f(|x-y|)$ is assumed to possess a finite variance, represented

by a Gaussian:

$$f_G(x) = \frac{1}{2\sqrt{\pi D_G}} \exp\left(-\frac{x^2}{4D_G}\right). \quad (7)$$

The class of processes with infinite second moment is represented by the Cauchy distribution:

$$f_C(x) = \frac{D_C}{\pi(D_C^2 + x^2)}. \quad (8)$$

In the latter case, realizations of the process are Lévy flights, whereas in the former diffusive walks occur.

The relaxation properties of the system are determined by the eigenvalue spectrum of $\rho(x|y)$. For $\rho(x|y)$ defined by Eq. (4) we find $0 \leq \lambda_i \leq 1$. The convergence time T of $p(x, t)$ is related to the largest non-unity eigenvalue $\lambda < 1$:

$$T = -[\log(\lambda)]^{-1}. \quad (9)$$

For constant $s(x)$ Eq. (9) reduces to Eq. (3).

We compare the Gaussian and Cauchian system in terms of their relaxation times T_G and T_C determined by Eq. (9). To this end, we let $D_G = (L/2\pi)D_C$. This gauge of the parameters D_G and D_C ensures that both systems require an equal amount of time ($T_G = T_C$) to reach the stationary state in a constant salience field.

3. Results

We generated ensembles of 10 000 salience fields at various values β . For each individual salience field we computed the convergence time of the Gaussian and the Cauchian system. Properties of the computed convergence time histograms are depicted in the lower left corner of Fig. 3. Irrespective of the second-order statistics of the salience field ensemble and the entire range of β the Cauchian system needs less time on average to reach the stationary state. The increase in T_C in response to a variable salience field is less pronounced than in T_G . The Cauchian system is more robust against variations in the salience field ensemble, the typical range of values for T_C is orders of magnitude more narrow than the range of T_G .

The temporal optimization arguments presented suggest that natural scanpaths are geometrically similar to model scanpaths (generated by means of Eq. (4)) if one chooses a function $f(|x - y|)$ with an algebraic tail. On the right in Fig. 3 model scanpaths of both, the Gaussian and Cauchian system, are shown. Here we chose the experimentally measured density of fixations (Fig. 1) as the salience field. The predicted similarity mentioned above is particularly obvious in the simulated model scanpaths. The Cauchian scanpath (bottom right in Fig. 3) looks strikingly similar to the experimentally observed paths of Fig. 1, whereas the Gaussian scanpath has no resemblance to natural scanpaths at all.

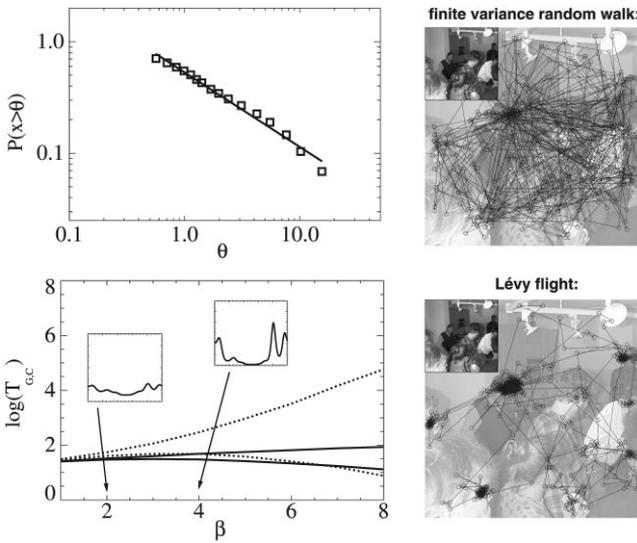


Fig. 3. Upper left: Probability $P(x > \theta)$ of generating a saccade of size $x > \theta$. On a double-logarithmic scale experimentally estimated $P(x > \theta)$ (denoted by squares) is well fitted by a line, implying a power law in the saccadic magnitude distribution. Lower left: Convergence times T_G (dotted lines) and T_C (solid lines) as functions of β . Typical values fall within the upper and lower curve in each case. The insets depict one-dimensional analogues of salience fields at two values of β (2 and 4). Upper right: Model scanpath generated by a finite variance random walk (Gaussian system) in an experimentally determined salience field. Bottom right: Same as above, except here the scanpath is a Lévy flight (Cauchian system).

We finally tested the validity and predictions of our theory by carrying out experiments on subjects scanning natural images on a computer screen similar to the one depicted in Fig. 1. The experiments were specifically designed to measure saccadic magnitude distributions under natural conditions. The results are shown in the upper left corner of Fig. 3. On a double-logarithmic scale the probability of generating a saccade of size $x > \theta$ is well fitted by a line, implying a power law in the saccadic magnitude distribution.

4. Discussion

We introduced a simple phenomenological stochastic model for human scanpath generation. The relevant quantity in our analysis is the relaxation time T of the process.

Our theory predicts that scanpaths generated under natural circumstances are similar in their nature to Lévy flights, i.e. possess a power law dependency in their magnitude distribution. The predictions were based on the plausible assumption that the visual system minimizes the time needed to scan the entire visual space. The power-law behavior predicted by the theory is in accordance with psychophysical experiments we carried out. Furthermore, the geometrical features of simulated

scanpaths with a power-law saccadic magnitude function are strikingly similar to those of natural scanpaths, whereas scanpaths with intrinsic scale are not.

The idea that the visual system adapts to an optimal sampling strategy of the visual environment is supported by an interesting set of experiments [5] on a subject who could not make eye movements. She compensated her oculomotoric deficit by rapid rotations of her head. Her visual perception though was normal and her scanpaths were similar to those produced by humans without deficit. Therefore, the possibility of a power law in saccadic magnitudes due to the specific mechanics of the human oculomotoric system can be ruled out.

The Markov assumption made in our model is supported by a recent experiment on covert visual attention. The covert visual attention system is thought to feed peripheral information to the saccadic system, and does not possess memory [6]. The Markov assumption can even be relaxed. Sufficiently, rapidly decaying correlations in successive gaze shifts do not alter the results presented here.

We conclude that a visual system producing Lévy flights implements an efficient strategy of shifting gaze in a random visual environment than any strategy employing a typical scale in gaze shift magnitudes. Our study provides an understanding of the ecology of scanpath generation observed in humans under natural conditions.

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Dirk Brockmann was born in Braunschweig, Germany. He studied Physics at Duke University, Durham, USA and the University of Göttingen, Germany where he received his “Diplomo” in the Department of Theoretical Physics in 1995. Since then he has been working in the Department of Nonlinear Dynamics at the Max-Planck-Institut für Strömungsforschung, Göttingen. Dirk is looking forward to receive his Ph.D. in Theoretical Physics in the year 2000. He was primarily interested in random walks, anomalous diffusion, the brain and getting enough sleep.



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